

PROPERTIES OF CERTAIN PARTIAL DYNAMIC INTEGRODIFFERENTIAL EQUATIONS

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ABSTRACT. The aim of the present paper is to study the existence, uniqueness and some other properties of solutions of a certain partial dynamic integrodifferential equations. The Banach fixed point theorem and certain fundamental inequality with explicit estimates are used to establish our results.

1. INTRODUCTION

The study of time scale calculus was initiated by Stefan Hilger in his Ph.D dissertation which unifies the continuous and discrete calculus[4]. Since then many authors have worked on various aspects dynamic equations on timescale calculus[5, 6, 7, 8, 9]. Basic information on time scale calculus can be found in [1, 2, 3, 4]. Many authors have studied various types of partial dynamic equations on time scales[7, 8, 10, 11, 14]. In [12, 13, 15] have studied the integrodifferential equations and its properties. Motivated by the results in the above papers in this paper we study properties of certain partial dynamic integrodifferential equations. In what follows \mathbb{R} denotes the set of real numbers and \mathbb{T} denotes the arbitrary time scales. Now we give some basic definitions of time scale calculus. The function $f : \mathbb{T} \rightarrow \mathbb{R}$ is said to be rd-continuous if f is continuous at each right dense point of \mathbb{T} and is denoted by C_{rd} . Let two time scales with at least two point be denoted by \mathbb{T}_1 and \mathbb{T}_2 and $\Omega = \mathbb{T}_1 \times \mathbb{T}_2$. The delta partial derivative of a real valued function f on $\mathbb{T}_1 \times \mathbb{T}_2$ has a Δ_1 partial derivative $f^{\Delta_1}(t_1, t_2)$ with respect to t_1 if for each $\epsilon > 0$ there exists a neighbourhood U_{t_1} of t_1 such that.

$$|f(\sigma_1(t_1), t_2) - f(s, t_2) - f^{\Delta_1}(t_1, t_2)(\sigma_1(t_1) - s)| \leq \epsilon |\sigma_1(t_1) - s|$$

for all $s \in U_{t_1}$. The delta partial derivative of a real valued function f on $\mathbb{T}_1 \times \mathbb{T}_2$ has a Δ_2 partial derivative $f^{\Delta_2}(t_1, t_2)$ with respect to t_2 if for each $\eta > 0$ there exists a neighbourhood U_{t_2} of t_2 such that

$$|f(t_1, \sigma_2(t_2)) - f(t_1, l) - f^{\Delta_2}(t_1, t_2)(\sigma_2(t_2) - l)| \leq \eta |\sigma_2(t_2) - l|$$

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for all $u \in U_{t_1}$.

The partial derivative of $w(x, y)$ for $(x, y) \in \Omega$ with respect to x, y and xy is denoted by $w^{\Delta_1}(x, y)$, $w^{\Delta_2}(x, y)$ and $w^{\Delta_1\Delta_2}(x, y) = w^{\Delta_2\Delta_1}(x, y)$. Suppose $I = [a, b]$ with $a < b$ and $\overline{\Omega} = \Omega \times I$. The partial derivative of $u(x, y, z)$ for $(x, y, z) \in C_{rd}(\overline{\Omega}, R)$ with respect to x, y and xy is defined by $w^{\Delta_1}(x, y, z)$, $w^{\Delta_2}(x, y, z)$ and $w^{\Delta_1\Delta_2}(x, y, z) = w^{\Delta_2\Delta_1}(x, y, z)$.

In this paper we study the partial dynamic integrodifferential equation of the form

$$u^{\Delta_2\Delta_1}(x, y, z) = F(x, y, z, u(x, y, z), u^{\Delta_1}(x, y, z), u^{\Delta_2}(x, y, z), (Hu)(x, y, z)), \quad (1.1)$$

with the conditions

$$u(x, y_0, z) = \alpha(x, z), \quad u(x_0, y, z) = \beta(y, z) \quad (1.2)$$

for $(x, y) \in \Omega$ where

$$(Hu)(x, y, z) = \int_a^b G(x, y, z, q, u(x, y, q), u^{\Delta_1}(x, y, q), u^{\Delta_2}(x, y, q)) \Delta q, \quad (1.3)$$

where $G \in C_{rd}(\overline{\Omega} \times \mathbb{R}^3, \mathbb{R})$, $F \in C_{rd}(\overline{\Omega} \times \mathbb{R}^4, \mathbb{R})$ and $\alpha, \beta \in C_{rd}(\mathbb{R}_+ \times I, \mathbb{R})$.

We have $u(x_0, y_0, z) = \alpha(x_0, z) = \beta(x_0, z)$.

Now for $u, u^{\Delta_1}, u^{\Delta_2} \in C_{rd}(\overline{\Omega}, \mathbb{R})$, we denote

$$|u(x, y, z)|_W = |u(x, y, z)| + |u^{\Delta_1}(x, y, z)| + |u^{\Delta_2}(x, y, z)|. \quad (1.4)$$

Let S be the space function satisfying the condition

$$|u(x, y, z)|_W = O(e_\lambda(x, y, |z|)), \quad (1.5)$$

where $\lambda > 0$ is a positive constant. In space S we define norm u by

$$|u|_s = \sup_{(x, y, z) \in \Omega \times I} [|u(x, y, z)|_w e_{\Theta\lambda}(x, y, |z|)]. \quad (1.6)$$

The norm defined (1.6) is clearly a Banach Space.

Then (1.5) implies that there is a constant $N \geq 0$ such that

$$|u(x, y, z)|_w \leq N(e_\lambda(x, y, |z|)), \quad (1.7)$$

and we have

$$|u|_s \leq N. \quad (1.8)$$

The solution of (1.1) and (1.2) is a function $u(x, y, z) \in C_{rd}(\overline{\Omega}, \mathbb{R}^n)$ satisfying (1.1) and (1.2). It is easy to see that $u(x, y, z)$ with (1.1) and

(1.2) satisfy the following dynamic integrodifferential equation.

$$\begin{aligned}
 & u(x, y, z) \\
 &= \alpha(x, z) + \beta(y, z) - \alpha(0, z) \\
 &+ \int_{x_0}^x \int_{y_0}^y F(s, t, z, u(s, t, z), u^{\Delta_1}(s, t, z), u^{\Delta_2}(s, t, z), (Hu)(s, t, z)) \Delta t \Delta s,
 \end{aligned} \tag{1.9}$$

for $(x, y, z) \in C_{rd}(\overline{\Omega}, \mathbb{R})$

$$\begin{aligned}
 & u^{\Delta_1}(x, y, z) \\
 &= \alpha^{\Delta_1}(x, z) \\
 &+ \int_{y_0}^y F(x, t, z, u(x, t, z), u^{\Delta_1}(x, t, z), u^{\Delta_2}(x, t, z), (Hu)(x, t, z)) \Delta t,
 \end{aligned} \tag{1.10}$$

$$\begin{aligned}
 & u^{\Delta_2}(x, y, z) \\
 &= \beta^{\Delta_2}(y, z) \\
 &+ \int_{x_0}^x F(s, y, z, u(s, y, z), u^{\Delta_1}(s, y, z), u^{\Delta_2}(s, y, z), (Hu)(s, y, z)) \Delta s.
 \end{aligned} \tag{1.11}$$

We need following Lemma given in [3].

Lemma [[3], Theorem 2.6] Let $u \in C_{rd}(\mathbb{T}, \mathbb{R}_+)$, $a \in \mathbb{R}_+$

$$u^{\Delta}(t) \leq a(t)u(t),$$

for all $t \in \mathbb{T}^k$, then

$$u(t) \leq u(t_0)e_a(t, t_0),$$

for all $t \in \mathbb{T}^k$.

2. MAIN RESULTS

Now we give our main results

Theorem 1.1 Suppose that the functions F, G in (1.1) satisfy the condition

$$\begin{aligned}
 & |F(x, y, z, u_1, u_2, u_3, u_4) - F(x, y, z, \overline{u}_1, \overline{u}_2, \overline{u}_3, \overline{u}_4)| \\
 & \leq M(x, y, z) [|u_1 - \overline{u}_1| + |u_2 - \overline{u}_2| + |u_3 - \overline{u}_3| + |u_4 - \overline{u}_4|], \quad (2.1)
 \end{aligned}$$

$$|G(x, y, z, q, u_1, u_2, u_3) - G(x, y, z, q, \overline{u}_1, \overline{u}_2, \overline{u}_3)|$$

$$\leq K(x, y, z, q) [|u_1 - \overline{u_1}| + |u_2 - \overline{u_2}| + |u_3 - \overline{u_3}|], \quad (2.2)$$

where $M \in C_{rd}(\overline{\Omega}, \mathbb{R}_+)$ and $K \in C_{rd}(\overline{\Omega} \times I, \mathbb{R}_+)$.

For λ as in (1.5), there exists a nonnegative γ_i ($i = 1, 2, 3$) such that

$$\begin{aligned} & \int_{x_0}^x \int_{y_0}^y M(s, t, z) [e_\lambda(s, t, |z|) \\ & + \int_a^b k(s, t, z, q) e_\lambda(s, t, |q|) \Delta q] \Delta t \Delta s \leq \gamma_1 e_\lambda(x, y, |z|), \end{aligned} \quad (2.3)$$

$$\begin{aligned} & \int_{y_0}^y M(x, t, z) [e_\lambda(x, t, |q|) \\ & + \int_a^b k(x, t, z, q) e_\lambda(x, t, |q|) \Delta q] \Delta t \leq \gamma_2 e_\lambda(x, y, |z|), \end{aligned} \quad (2.4)$$

$$\begin{aligned} & \int_{x_0}^x M(s, y, z) [e_\lambda(x, t, |z|) \\ & + \int_a^b k(s, y, z, q) e_\lambda(s, y, |q|) \Delta q] \Delta s \leq \gamma_3 e_\lambda(x, y, |z|), \end{aligned} \quad (2.5)$$

for $x, y \in \Omega$, $z \in I$.

There exist nonnegative constants η_i ($i = 1, 2, 3$) such that

$$\begin{aligned} & |\alpha(x, z)| + |\beta(y, z)| + |\alpha(0, z)| \\ & + \int_{x_0}^x \int_{y_0}^y |f(s, t, z, 0, 0, (H0)(s, t, z))| \Delta t \Delta s \leq \eta_1 e_\lambda(x, y, |z|), \end{aligned} \quad (2.6)$$

$$|\alpha^\Delta(x, z)| + \int_{y_0}^y |F(x, t, z, 0, 0, 0, (H0)(x, t, z))| \Delta t \leq \eta_2 e_\lambda(x, y, |z|), \quad (2.7)$$

$$|\beta^\Delta(y, z)| + \int_{y_0}^y |F(s, y, z, 0, 0, 0, (H0)(s, y, z))| \Delta s \leq \eta_3 e_\lambda(x, y, |z|), \quad (2.8)$$

where α, β are as in (1.2).

If $\gamma = \gamma_1 + \gamma_2 + \gamma_3 < 1$ then problem (1.1) – (1.2) has a unique solution $u(x, y, z)$ on (1.1) – (1.2) in S .

Proof. Let $u(x, y, z) \in S$ and define the operator \mathbb{T} by

$$\begin{aligned} & (Tu)(x, y, z) \\ &= \alpha(x, z) + \beta(y, z) - \alpha(0, z) \\ &+ \int_{x_0}^x \int_{y_0}^y F(s, t, z, u(s, t, z), u^{\Delta_1}(s, t, z), u^{\Delta_2}(s, t, z), (Hu)(s, t, z)) \Delta t \Delta s. \end{aligned} \quad (2.9)$$

Now we show that P maps S into itself. Tu is rd-continuous on $\Omega \times I$ and $Tu \in R$.

From (2.9) and given hypotheses we have

$$\begin{aligned} & (Pu)(x, y, z) \\ & \leq |\alpha(x, z)| + |\beta(y, z)| + |\alpha(0, z)| \\ & + \int_{x_0}^x \int_{y_0}^y |F(s, t, z, u(s, t, z), u^{\Delta_1}(s, t, z), u^{\Delta_2}(s, t, z), (Hu)(s, t, z)) \\ & - F(s, t, z, 0, 0, 0, (H0)(s, t, z))| \Delta t \Delta s \\ & + \int_{x_0}^x \int_{y_0}^y |F(s, t, z, 0, 0, 0, (H0)(s, t, z))| \Delta t \Delta s \\ & \leq \eta_1 e_\lambda(x, y, |z|) + \int_{x_0}^x \int_{y_0}^y M(s, t, z) [e_\lambda(s, t, |z|) |u(s, t, z)| e_{\Theta\lambda}(s, t, |z|) \\ & + \int_a^b k(x, y, z, q) e_\lambda(s, t, |q|) |u(x, y, |q|)|_W e_{\Theta\lambda}(s, t, |q|) \Delta q] \Delta t \Delta s \\ & \leq \eta_1 e_\lambda(x, y, |z|) + |u|_s \int_{x_0}^x \int_{y_0}^y M(s, t, z) [e_\lambda(s, t, |z|) \\ & + \int_a^b k(x, y, z, q) e_\lambda(s, t, |q|) \Delta q] \Delta t \Delta s \\ & \leq [\eta_1 + N\gamma_1] e_\lambda(x, y, |z|). \end{aligned} \quad (2.10)$$

Delta differentiating on both sides of (2.9) with respect to x and (1.8) we have

$$\begin{aligned}
& \left| (Pu)^{\Delta_1}(x, y, z) \right| \\
& \leq \alpha^{\Delta_1}(x, z) \\
& + \int_{y_0}^y \left| F(x, t, z, u(x, t, z), u^{\Delta_1}(x, t, z), u^{\Delta_2}(x, t, z), (Hu)(x, t, z)) \right. \\
& \quad \left. - F(x, t, z, 0, 0, 0, (H0)(x, t, z)) \right| \Delta t \\
& + \int_{y_0}^y \left| F(x, t, z, 0, 0, 0, (H0)(x, t, z)) \right| \Delta t \\
& \leq \eta_2 e_\lambda(x, y, |z|) + |u|_s \int_{y_0}^y M(x, t, z) [e_\lambda(x, t, |z|) \\
& \quad + \int_a^b k(x, t, z, q) e_\lambda(x, t, |q|) \Delta q] \Delta t \\
& \leq [\eta_2 + N\gamma_2] e_\lambda(x, y, |z|). \tag{2.11}
\end{aligned}$$

Similarly we have

$$\left| (Pu)^{\Delta_2}(x, y, z) \right| \leq [\eta_3 + N\gamma_3] e_\lambda(x, y, |z|). \tag{2.12}$$

From (2.10) – (2.12) we have

$$|Pu|_s \leq [(\eta_1 + \eta_2 + \eta_3) + N\gamma].$$

Thus proving that P maps S into itself.

Now we show that operator P is a contraction map. Let $u(x, y, z), \bar{u}(x, y, z) \in S$. From (2.9) we have

$$\begin{aligned}
& |(Pu)(x, y, z) - (P\bar{u})(x, y, z)| \\
& \leq \int_{x_0}^x \int_{y_0}^y \left| F(s, t, z, u(s, t, z), u^{\Delta_1}(s, t, z), u^{\Delta_2}(s, t, z), (Hu)(s, t, z)) \right. \\
& \quad \left. - F(s, t, z, \bar{u}(s, t, z), \bar{u}^{\Delta_1}(s, t, z), \bar{u}^{\Delta_2}(s, t, z), (H\bar{u})(s, t, z)) \right| \Delta t \Delta s \\
& \leq |u - \bar{u}|_s \int_{x_0}^x \int_{y_0}^y M(x, t, z) [e_\lambda(s, t, |z|)
\end{aligned}$$

$$\begin{aligned}
 & + \int_a^b k(s, t, z, q) e_\lambda(s, t, |q|) \Delta q \Big] \Delta t \Delta s \\
 & \leq |u - \bar{u}|_s \gamma_1 e_\lambda(x, y, |z|). \tag{2.13}
 \end{aligned}$$

Similarly delta differentiating both sides of (2.12) with respect to x and y we have

$$\left| (Pu)^{\Delta_1}(x, y, z) - (P\bar{u})^{\Delta_1}(x, y, z) \right| \leq |u - \bar{u}|_s \gamma_2 e_\lambda(x, y, |z|), \tag{2.14}$$

and

$$\left| (Pu)^{\Delta_2}(x, y, z) - (P\bar{u})^{\Delta_2}(x, y, z) \right| \leq |u - \bar{u}|_s \gamma_3 e_\lambda(x, y, |z|). \tag{2.15}$$

From (2.13) – (2.15) we obtain

$$|Pu - P\bar{u}|_s \leq \gamma |u - \bar{u}|_s.$$

Since $\gamma < 1$, P has a unique fixed point in S by Banach fixed point theorem. The fixed point of P is a solution of (1.1) – (1.2). This completes the proof.

3. PROPERTIES OF SOLUTIONS

Now we study the properties of solution of dynamic integrodifferential equation of the form

$$u^{\Delta_2 \Delta_1}(x, y, z) = f(x, y, z, u(x, y, z), (hu)(x, y, z)), \tag{3.1}$$

with (1.2) for $(x, y, z) \in \bar{\Omega}$ where

$$(hu)(x, y, z) = \int_a^b j(x, y, z, q, u(x, y, q)) dq, \tag{3.2}$$

in which $i \in C_{rd}(\bar{\Omega} \times \mathbb{R}, \mathbb{R})$, $f \in C_{rd}(\Omega \times \mathbb{R}^2, \mathbb{R})$.

Now we prove the following dynamic inequality which can be used in studying some properties of solutions.

Theorem 3.1 Let $w, p \in C_{rd}(\bar{\Omega}, \mathbb{R}_+)$, $\in C_{rd}(\bar{\Omega} \times I, \mathbb{R}_+)$ and $c \geq 0$ a constant. If

$$\begin{aligned}
 w(x, y, z) & \leq c + \int_{x_0}^x \int_{y_0}^y [p(s, t, z) w(s, t, z) \\
 & + \int_a^b r(s, t, z, q) w(s, t, q) \Delta q] \Delta t \Delta s, \tag{3.3}
 \end{aligned}$$

for $(x, y, z) \in \Omega$ then

$$w(x, y, z) \leq ce_{Q(x,y,z)}(x, x_0), \quad (3.4)$$

where $(x, y, z) \in \overline{\Omega}$ and

$$Q(x, y, z) = \int_{y_0}^y \left[p(s, t, Z) \int_a^b r(s, t, Z, q) \Delta q \right] \Delta s. \quad (3.5)$$

Proof. For an arbitrary $Z \in I$ from (3.3) we have

$$\begin{aligned} w(x, y, Z) \leq c + \int_{x_0}^x \int_{y_0}^y [p(s, t, Z) w(s, t, Z) \\ + \int_a^b r(s, t, Z, q) w(s, t, q) \Delta q] \Delta t \Delta s. \end{aligned} \quad (3.6)$$

Put

$$m(s, t) = p(s, t, Z) w(s, t, Z) + \int_a^b r(s, t, z, q) w(s, t, q) \Delta q. \quad (3.7)$$

The inequality (3.6) becomes

$$w(x, y, Z) \leq c + \int_{x_0}^x \int_{y_0}^y m(s, t) \Delta t \Delta s. \quad (3.8)$$

Now define

$$v(x, y) = c + \int_{x_0}^x \int_{y_0}^y m(s, t) \Delta t \Delta s, \quad (3.9)$$

then

$$v(0, y) = v(x, 0) = c, w(x, y, Z) \leq v(x, y). \quad (3.10)$$

Delta differentiating both sides of (3.9) with respect to x and y using (3.7) and (3.10) we have

$$\begin{aligned} v^{\Delta_2 \Delta_1}(x, y) &= m(x, y) \\ &= p(x, y, Z) w(x, y, Z) + \int_a^b r(x, y, Z, q) w(x, y, q) \Delta q \end{aligned}$$

$$\leq v(x, y) \left[p(x, y, Z) + \int_a^b r(s, t, Z, q) \Delta q \right]. \quad (3.11)$$

By keeping x fixed in (3.11), and taking $y = t$ and delta integrating with respect to second variable from y_0 to y . Using the fact that

$$\begin{aligned} v^{\Delta_1}(x, y) &\leq \int_{y_0}^y \left[p(x, t, Z) + \int_a^b r(x, t, Z, q) \Delta q \right] v(x, t) \Delta t \\ &\leq v(x, y) \int_{y_0}^y \left[p(x, t, Z) + \int_a^b r(x, t, Z, q) \Delta q \right] \Delta t \\ &\leq v(x, y) Q(x, y, Z). \end{aligned} \quad (3.12)$$

Now treating y fixed in (3.12) and applying Lemma we have

$$v(x, y) \leq ce_{Q(x, y, Z)}(x, x_0). \quad (3.13)$$

Because Z is arbitrary and using (3.10) we get (3.9).

Theorem 3.2 Suppose the functions f, j in (3.1), (3.2) satisfy the conditions

$$|f(x, y, z, u, v) - f(x, y, z, \bar{u}, \bar{v})| \leq p_1(x, y, z) [|u - \bar{u}| + |v - \bar{v}|], \quad (3.14)$$

$$|j(x, y, z, q, u) - j(x, y, z, \bar{q}, \bar{u})| \leq p_2(x, y, z, q) |u - \bar{u}|, \quad (3.15)$$

where $p_1 \in C_{rd}(\bar{\Omega}, \mathbb{R}_+)$, $p_2 \in C_{rd}(\bar{\Omega} \times I, \mathbb{R}_+)$, $c \geq 0$ and

$$\int_{x_0}^x \int_{y_0}^y \left[p_1(s, t, z) + \int_a^b p_2(s, t, z, q) \Delta q \right] \Delta t \Delta s < \infty, \quad (3.16)$$

then the problem (3.1) – (1.1) has at most one solution.

Proof. Let $u_1(x, y, z)$ and $u_2(x, y, z)$ be two solutions of problem (3.1) – (1.1).

$$\begin{aligned} &|u_1(x, y, z) - u_2(x, y, z)| \\ &\leq \int_{x_0}^x \int_{y_0}^y |f(s, t, z, u_1(s, t, z), (hu_1)(s, t, z)) \\ &\quad - f(s, t, z, u_2(s, t, z), (hu_2)(s, t, z))| \Delta t \Delta s \\ &\leq \int_{x_0}^x \int_{y_0}^y [p_1(s, t, z) |u_1(s, t, z) - u_2(s, t, z)| \end{aligned}$$

$$\begin{aligned}
& + |(hu_1)(s, t, z) - (hu_2)(s, t, z)| \Delta t \Delta s \\
& \leq \int_{x_0}^x \int_{y_0}^y [p_1(s, t, z) |u_1(s, t, z) - u_2(s, t, z)| \\
& + \int_a^b p_1(s, t, z, q) |u_1(s, t, q) - u_2(s, t, q)| \Delta q] \Delta t \Delta s. \quad (3.17)
\end{aligned}$$

Now applying Theorem 3.1 to (3.17) yields $|u_1(x, y, z) - u_2(x, y, z)| \leq 0$ which gives $u_1(x, y, z) = u_2(x, y, z)$. This proves that there is at most one solution to problem (3.1) – (1.1).

Now we prove the theorem which gives the boundedness of solution of (3.1) – (1.1).

Theorem 3.3. Suppose the function f, j, α, β in (3.1) – (1.1) satisfy the conditions

$$|f(x, y, z, u, v)| \leq p_1(x, y, z) [|u| + |v|], \quad (3.18)$$

$$|j(x, y, z, u, v)| \leq p_2(x, y, z, q) |u|, \quad (3.19)$$

$$|\alpha(x, z) + \beta(y, z) - \alpha(0, z)| \leq c, \quad (3.20)$$

where $p_1 \in C_{rd}(\Omega, \mathbb{R}_+)$, $p_2 \in C_{rd}(\Omega \times I, \mathbb{R}_+)$, $c \geq 0$ is a constant and the condition (3.16) holds. Then solution $u(x, y, z)$ is bounded and

$$|u(x, y, z)| \leq ce_{Q(x, y, z)}(x, x_0), \quad (3.21)$$

for $(x, y, z) \in \overline{\Omega}$

Proof. Since $u(x, y, z)$ is a solution of (3.1) – (1.1). We have

$$\begin{aligned}
& |u(x, y, z)| \leq |\alpha(x, z) + \beta(y, z) - \alpha(0, z)| \\
& + \int_{x_0}^x \int_{y_0}^y |f(s, t, z, u(s, t, z), (hu)(s, t, z))| \Delta t \Delta s \\
& \leq c + \int_{x_0}^x \int_{y_0}^y [p_1(s, t, z) |u(s, t, z)| \\
& + \int_a^b p_2(s, t, z, q) |u(s, t, q)| \Delta q] \Delta t \Delta s. \quad (3.22)
\end{aligned}$$

Now an application of Theorem 3.1 to (3.22) yields (3.21) thus proving the boundedness of solution.

Now we give the dependency of solution of equation on given condition

Theorem 3.4. Suppose the function f, k in (3.1), (3.2) satisfy the conditions (3.14), (3.15) and the condition (3.16) holds. Let $u(x, y, z)$ and $v(x, y, z)$ be the solutions of equation with condition (1.2) and

$$v(x, 0, z) = \bar{\alpha}(x, z), \quad v(0, y, z) = \bar{\beta}(y, z), \quad (3.23)$$

respectively and

$$|\alpha(x, z) + \beta(y, z) - \alpha(0, z) - [\bar{\alpha}(x, z) + \bar{\beta}(y, z) - \bar{\alpha}(0, z)]| \leq a, \quad (3.24)$$

where $\alpha, \beta, \bar{\alpha}, \bar{\beta} \in C_{rd}(\mathbb{R}_+ \times I, \mathbb{R})$ and $a \geq 0$ is constant. Then

$$|u(x, y, z) - v(x, y, z)| \leq ae_{Q(x, y, z)}(x, x_0). \quad (3.25)$$

Proof. Since $u(x, y, z)$ and $v(x, y, z)$ are solutions of (3.1)-(1.1) and (3.1) – (3.23) and the given conditions we have

$$\begin{aligned} & |u(x, y, z) - v(x, y, z)| \\ & \leq |\alpha(x, z) + \beta(y, z) - \alpha(0, z) \\ & \quad - [\bar{\alpha}(x, z) + \bar{\beta}(y, z) - \bar{\alpha}(0, z)]| \\ & + \int_{x_0}^x \int_{y_0}^y |f(s, t, z, u(s, t, z), (hu)(s, t, z)) \\ & \quad - f(s, t, z, v(s, t, z), (hu)(s, t, z))| \Delta t \Delta s \\ & \leq a + \int_{x_0}^x \int_{y_0}^y [p_1(s, t, z) |u(s, t, z) - v(s, t, z)| \\ & \quad + \int_a^b p_2(s, t, z, q) |u(s, t, z) - v(s, t, z)| \Delta q] \Delta t \Delta s. \end{aligned} \quad (3.26)$$

Now an application of Theorem 3.1 to (3.26) gives the estimate (3.25) which gives the dependency of solution of equation (3.1) on given conditions.

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